PROOF OF $e^{i\theta} = \cos\theta + i\sin\theta$

This proof was discovered by Euler in 1748. Today it is indispensable in topics such as AC circuits and damped harmonic motion.

We know:

$$e^{x} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{6!} + \ldots$$

Therefore,

$$e^{i\theta} = \frac{(i\theta)^0}{0!} + \frac{(i\theta)^1}{1!} + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \frac{(i\theta)^6}{6!} + \frac{(i\theta)^7}{7!} + \ldots$$

Since $i = \sqrt{-1}$,

$$e^{i\theta} = 1 + \frac{i\theta}{1!} - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} + \frac{i\theta^5}{5!} - \frac{\theta^6}{6!} - \frac{i\theta^7}{7!} \ldots$$

$$= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots\right) + i\left(\frac{\theta}{1!} - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots\right)$$

And we know that $1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots = \cos\theta$ and $\frac{\theta}{1!} - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots = \sin\theta$. Therefore,

$$e^{i\theta} = \cos\theta + i\sin\theta$$

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HERE'S SOMETHING ELSE ÜBER COOL

The five most important numbers in mathematics in one equation:

$$e^{i\pi} + 1 = 0$$

Source

Mr. "Why-Math-is-My-Life" McMaster, Personal Communication, June 2004